JEE(ADVANCED) - 2013 PAPER 1

MATHEMATICS

[Time allowed: 3 hours] **[Maximum Marks: 180]**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose

INSTRUCTIONS

A. General

1. This booklet is your Question paper. Do not break the seats of his booklet before being instructed to do so by the invigilators.
2. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.
3. Write your name and roll number in the space provided on the back cover of this booklet.
4. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination
5. **Using a black ball point pen, darken the bubbles on the upper original sheet**. Apply sufficient pressure so that the impression is created on the bottom sheet.

B. Question Paper Format

This question paper consists three sections.
Section 1 contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct**.
Section 2 contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct.**
Section 3 contains **5 questions**. The answer to each question is a single- digit integer, ranging from 0 to 9 (both inclusive).

C. Marking Scheme

For each question **Section 1**, you will be awarded **2 marks** if you darken the bubble corresponding to only the correct answer(s) and zero mark if no bubble are darkened. In all other cases, minus one (-1) mark will be awarded
For each question **Section 2**, you will be awarded **4 marks** if you darken the bubble corresponding to only the correct answer(s) and zero mark if no bubble are darkened. In all other cases, minus one (-1) mark will be awarded
For each question **Section 3**, you will be awarded **4 marks** if you darken the bubble corresponding to only the correct answer(s) and zero mark if no bubble are darkened. In all other cases, minus one (-1) mark will be awarded

SECTION - 1:

(One or more option correct Type)

Q.41. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{x+1}{-1}=\frac{z}{3}$ to the plane $x+y+z=3$. The feet of perpendiculars lie on the line

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$

(B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

(C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$

(D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

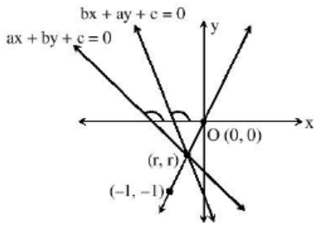
Q 42. For a $a>b>c>0$, the distance between $(1,1)$ and the point of intersection of the lines $ax+by+c=0$ and $bx+xy+c=0$ is less than $2\sqrt{2}$, then

(A) $a+b-c>0$

(B) $a-b+c<0$

(C) $a-b+c>0$

(D) $a+b-c<0$



Q.43. The area enclosed by the curves $y = \sin x + l\cos x \, and \, y = l\cos x - \sin xl$ over the interval $\left[0, \frac{\pi}{2}\right]$

(A) $4\left(\sqrt{2}-1\right)$

(B) $2\sqrt{2}\left(\sqrt{2}-1\right)$

(C) $2\left(\sqrt{2}-1\right)$

(D) $2\sqrt{2}\left(\sqrt{2}+1\right)$

Q.44. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is

(A) $\frac{235}{256}$

(B) $\frac{21}{256}$

(C) $\frac{3}{256}$

(D) $\frac{253}{256}$

*Q. 45. Let complex numbers α and $\frac{\alpha}{1}$ lie on circles $(x-x_0)^2 + (y-y_0)^2 = r^2 \, and \, (x-x_0)^2 + (y-y_0)^2 = 4r^2$,respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$. then $|\alpha| =$

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{7}}$

(D) $\frac{1}{3}$

Q. 46. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is

(A) 6

(B) 4

(C) 2

(D) 0

Q. 47. Let $f:\left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval

(A) $(2e-1, 2e)$

(B) $(e-1, 2e-1)$

(C) $\left(\frac{e-1}{2}, e-1\right)$

(D) $\left(0, \frac{e-1}{2}\right)$

Q. 48. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ *and* $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $\overrightarrow{PT} = \hat{i} - 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$, and $\overrightarrow{PS}$ is

(A) 5

(B) 20

(C) 10

(D) 30

Q. 49. The value of $\cot\left(\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right)$ is

(A) $\frac{23}{25}$

(B) $\frac{25}{23}$

(C) $\frac{23}{24}$

(D) $\frac{24}{23}$

Q. 50. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$ .Then the equation of the curve is

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$

(B) $\operatorname{cos}ec\left(\frac{y}{x}\right) = \log x + 2$

(C) $sec\left(\frac{2y}{x}\right)=\log x+2$

(D) $\cos\left(\frac{2y}{x}\right)=\log x+\frac{1}{2}$

SECTION — 2: (One or more options correct Type)

Q. 51. A line l passing through the origin is perpendicular to the lines
$l_1:(3+t)\hat{i}+(-+2t)\hat{j}+(4+2t)\hat{k},-\infty<t<\infty, l_2:(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k},-\infty<s<\infty$
+ $2t$ Then, the coordinate (s) of the point (s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_2 is

(A) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$

(B) $(-1,-1,0)$

(C) $(1, 1, 1)$

(D) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

Q. 52. Let $f(x)=x\sin \pi x, x>0$. Then for all natural numbers n, $f'(x)$ vanishes at

(A) a unique point in the interval $\left(n,\ n+\frac{1}{2}\right)$

(B) a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(C) a unique point in the interval $(n,\ n+1)$

(D) two points in the interval $(n,\ n+1)$

Q. 53. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k-1)}{2}} k^2$. Then S_n can take value(s)

(A) 1056

(B) 1088

(C) 1120

(D) 1332

Q. 54. For 3×3 3x3 matrices M and N , which of the following statement (s) is (are) NOT correct ?

(A) $N^T MN$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(B) $MN - NM$ is skew symmetric for all symmetric matrices M and N

(C) MN is symmetric for all symmetric matrices M and N

(D) $(adj\, M)(adj\, N) = adj\,(MN)$ for all invertible matrices M and N

Q. 55. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio $8:15$ is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100 , the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are

(A) 24

(B) 32

(C) 45

(D) 60

SECTION — 3: (Integer value correct Type)

Q. 56. Consider the set of eight vectors $V=\left\{a\hat{i}+b\hat{j}+c\hat{k};\ a,\ b,\ c\in\{-1,1\}\right\}$.Three non-coplanar vectors can be chosen from V in 2^p 2P ways. Then P is________

Q. 57. Of the three independent events $E_1,\ E_2$ and E_3, the probability that only E_1 occurs is α only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events $E_1,\ E_2\ or\ E_3$ occurs satisfy the equations $(\alpha-2\beta)p=\alpha\beta\ and\ (\beta-2\gamma)p=2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0,1)$.

Then $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}=-$

*Q. 58. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio $5:10:14$. Then $n=$ ________

*Q. 59. A pack contains n cards numbered from $1\ to\ n$. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k-20=$ ________

*Q. 60. A vertical line passing through the point $(h,0)$ the tangents to the ellipse $\frac{x^2}{4}+\frac{y^2}{4}=1$ at P and Q meet at the point R. If $\Delta(h)=$ area of the triangle

$PQR,\Delta_1=\max\limits_{1/2\le h\le 1}\Delta(h)\ and\ \Delta_2=\max\limits_{1/2\le h\le 1}\Delta(h),then\ \frac{8}{\sqrt{5}}\Delta_1-8\Delta_2=$________